$D_{s0}^*(2317)$ and $D_{s1}(2460)$ mesons in two-body B-meson decays

Amand Faessler[1], Thomas Gutsche[1], Sergey Kovalenko[2], Valery E. Lyubovitskij[1] *

[1] *Institut für Theoretische Physik, Universität Tübingen, Auf der Morgenstelle 14, D-72076 Tübingen, Germany*

[2] *Centro de Estudios Subatómicos(CES), Universidad Técnica Federico Santa María, Casilla 110-V, Valparaíso, Chile*

(Dated: February 1, 2008)

We analyze the branching ratios of $B \to D^{(*)} D_{s0}^*(D_{s1})$ decays using the factorization hypothesis. The $B \to D^{(*)}$ transition form factors are taken from a model-independent analysis done by Caprini, Lellouch and Neubert based on heavy quark spin symmetry and dispersive constraints, including short-distance and power corrections. The leptonic decay constants $f_{D_{s0}^*}$ and $f_{D_{s1}}$ are calculated assuming a molecular structure for the D_{s0}^* and D_{s1} mesons. The calculated branching ratios of B-meson two-body decays are compared with experimental data and other theoretical results.

PACS numbers: 13.25.Hw,13.20.Fc,13.40.Hq,14.40.Lb,14.40.Nd
Keywords: charm and bottom mesons, leptonic and nonleptonic decays, hadronic molecule

* On leave of absence from Department of Physics, Tomsk State University, 634050 Tomsk, Russia

I. INTRODUCTION

Presently there is strong interest to study a newly observed mesons and baryons in the context of a hadronic molecule interpretation (for overview see e.g. Ref. [1]). In the present work we focus on weak production properties of scalar $D^*_{s0}(2317)$ and axial $D_{s1}(2460)$ charm-strange mesons (for a review see e.g. [2]). The $D^*_{s0}(2317)$ meson was discovered just a few years ago by the BABAR Collaboration at SLAC in the inclusive $D_s^+\pi^0$ invariant mass distribution from e^+e^- annihilation data [3]. The nearby state $D_{s1}(2460)$ decaying into $D_s^*\pi^0$ was observed by the CLEO Collaboration at CESR [4]. Both of these states have been confirmed by the Belle Collaboration at KEKB [5]. In the interpretation of these experiments it was suggested that the $D^*_{s0}(2317)$ and $D_{s1}(2460)$ mesons are the P-wave charm-strange quark states with spin-parity quantum numbers $J^P = 0^+$ and $J^P = 1^+$, respectively. In the following the Belle [6] and BABAR [7] Collaborations observed the production of $D^*_{s0}(2317)$, $D_{s1}(2460)$ and their subsequent strong and radiative transitions in the nonleptonic two-body B decays. The most recent data of BABAR and Belle on two-body B-meson decays into $D^*_{s0}(2317)$ and $D_{s1}(2460)$ states can be found in Refs. [8, 9]. It is worth noting that the existing experimental information on the properties of $D^*_{s0}(2317)$ and $D_{s1}(2460)$ mesons [10] leaves quite a significant uncertainty in their interpretation as $J^P = 0^+$ and $J^P = 1^+$ states.

Theoretical analysis of $B \to D^{(*)}D^*_{s0}(D_{s1})$ decays has been performed in different approaches [11]-[20] based on the factorization hypothesis, which essentially simplifies the calculation of the transition amplitude. The factorizable amplitude $B \to D^{(*)}D^*_{s0}(D_{s1})$ is given by the product of the corresponding form factors (or their combination) describing semileptonic $\bar{B} \to D^{(*)}\ell\bar{\nu}_\ell$ transitions and the leptonic decay constant $f_{D^*_{s0}}$ $(f_{D_{s1}})$. The leptonic decay constants $f_{D^*_{s0}}$ and $f_{D_{s1}}$ have been calculated directly or extracted from the analysis of $B \to D^{(*)}D^*_{s0}(D_{s1})$ decays in Refs. [11, 12, 14, 17, 18], [20]-[25]. The form factors of $B \to D^{(*)}\ell\bar{\nu}_\ell$ transitions have been analyzed and calculated in various theoretical approaches such as: heavy quark effective theory, QCD sum rules, lattice QCD, different types of quark and soliton models, approaches based on the solution of Bethe-Salpeter and Faddeev equations, etc.

In this paper we assume the D^*_{s0} and D_{s1} mesons to be hadronic molecules - bound states of D, K and D^*, K mesons, respectively. Using this molecular picture, in Refs. [26, 27] we calculated strong and radiative decays of D^*_{s0} and D_{s1} mesons. The obtained results are in agreement with other theoretical approaches, e.g. the strong decay widths $D^*_{s0} \to D_s\pi^0$ and $D_{s1} \to D_s^*\pi^0$ are of the order of 10^2 KeV and the radiative decays $D^*_{s0} \to D_s^*\gamma$, $D_{s1} \to D_s\gamma$, etc. are of the order of a few KeV. Here, using the same approach, we calculate the leptonic decay constants $f_{D^*_{s0}}$ and $f_{D_{s1}}$. For the form factors governing the semileptonic $\bar{B} \to D^{(*)}\ell\bar{\nu}_\ell$ transitions we use the model-independent results obtained by Caprini, Lellouch and Neubert (CLN) [28] on the basis of heavy quark spin symmetry and dispersive constraints, including short-distance and power corrections. Note, that in Ref. [18] the authors already used the CLN results in their analysis of two-body $B \to DD^*_{s0}(D_{s1})$ transitions restricting themselves to the heavy quark limit and modes with a pseudoscalar D meson in the final state. Using the experimental lower limits for the $B \to DD^*_{s0}(D_{s1})$ branching ratios they derived lower limits for the products $|a_1|f_{D^*_{s0}}$ and $|a_1|f_{D_{s1}}$, where a_1 is a combination of the short-distance Wilson coefficients [29]-[31].

In the present paper we proceed as follows. First, in Section II, we discuss the basic notions of our approach. We indicate and evaluate the effective mesonic Lagrangian for the treatment of charmed mesons $D^*_{s0}(2317)$ and $D_{s1}(2460)$ as DK and D^*K bound states, respectively. Then in Section III we discuss the calculation of the leptonic decay constants $f_{D^*_{s0}}$ and $f_{D_{s1}}$. In Section IV we present a detailed analysis of two-body bottom meson decays $B \to D^{(*)}D^*_{s0}(D_{s1})$ applying the factorization hypothesis. As we already stressed before, in this analysis we use the model-independent CLN results [28] for the weak form factors defining the $B \to D^{(*)}\ell\bar{\nu}_\ell$ transitions. In Section V we give a short summary of our results.

II. MOLECULAR STRUCTURE OF $D^{*\pm}_{s0}(2317)$ AND $D^{\pm}_{s1}(2460)$ MESONS

In this section we discuss the formalism for the study of the $D^{*\pm}_{s0}(2317)$ and $D^{\pm}_{s1}(2460)$ mesons as hadronic molecules, represented by DK and D^*K bound states, respectively. We adopt that the isospin, spin and parity quantum numbers of $D^{*\pm}_{s0}(2317)$ and $D^{\pm}_{s1}(2460)$ are: $I(J^P) = 0(0^+)$ and $I(J^P) = 0(1^+)$, while for their masses we take the values: $m_{D^*_{s0}} = 2.3173$ GeV and $m_{D_{s1}} = 2.4589$ GeV [10]. Our framework is based on effective interaction Lagrangians describing the couplings of $D^*_{s0}(2317)$ and $D_{s1}(2460)$ mesons to their constituents:

$$\mathcal{L}_{D^*_{s0}}(x) \; = \; g_{_{D^*_{s0}}} \, D^{*\,-}_{s0}(x) \int dy \, \Phi_{D^*_{s0}}(y^2) \, D(x + w_{_{KD}} y) \, K(x - w_{_{DK}} y) \, + \, \text{H.c.} \,, \tag{1}$$

$$\mathcal{L}_{D_{s1}}(x) \; = \; g_{_{D_{s1}}} \, D^{\mu\,-}_{s1}(x) \int dy \, \Phi_{D_{s1}}(y^2) \, D^*_\mu(x + w_{_{KD^*}} y) \, K(x - w_{_{D^*K}} y) \, + \, \text{H.c.} \,, \tag{2}$$

where the doublets of $D^{(*)}$ and K mesons are defined as

$$D = \begin{pmatrix} D^0 \\ D^+ \end{pmatrix}, \qquad D^* = \begin{pmatrix} D^{*\,0} \\ D^{*\,+} \end{pmatrix}, \qquad K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}. \tag{3}$$

The summation over isospin indices is understood. The molecular structure of the $D_{s0}^{*\pm}$ and $D_{s1}^{\pm}$ states is:

$$\begin{aligned} |D_{s0}^{*+}\rangle &= |D^+K^0\rangle + |D^0K^+\rangle\,, \qquad & |D_{s0}^{*-}\rangle &= |D^-\bar{K}^0\rangle + |\bar{D}^0K^-\rangle\,, \\ |D_{s1}^{+}\rangle &= |D^{*+}K^0\rangle + |D^{*0}K^+\rangle\,, \qquad & |D_{s1}^{-}\rangle &= |D^{*-}\bar{K}^0\rangle + |\bar{D}^{*0}K^-\rangle\,. \end{aligned} \tag{4}$$

The correlation functions Φ_M with $M = D_{s0}^*$ or D_{s1} characterize the finite size of the $D_{s0}^*(2317)$ and $D_{s1}(2460)$ mesons as DK and D^*K bound states and depend on the relative Jacobi coordinate y with, in addition, x being the center of mass (CM) coordinate. Note, that the local limit corresponds to the substitution of Φ_M by the Dirac delta-function: $\Phi_M(y^2) \to \delta^4(y)$. In Eqs. (1) and (2) we introduced the kinematical parameters w_{ij}:

$$w_{ij} = \frac{m_i}{m_i + m_j}\,, \tag{5}$$

where m_D, m_{D^*} and m_K are the masses of D, D^* and K mesons. The Fourier transform of the correlation function reads

$$\Phi_M(y^2) \,=\, \int \frac{d^4p}{(2\pi)^4}\, e^{-ipy}\, \widetilde{\Phi}_M(-p^2)\,, \qquad M = D_{s0}^*, D_{s1}\,. \tag{6}$$

A basic requirement for the choice of an explicit form of the correlation function is that it falls down sufficiently fast in the ultraviolet region of Euclidean space to render the Feynman diagrams ultraviolet finite. We adopt the Gaussian form

$$\tilde{\Phi}_M(p_E^2) \doteq \exp(-p_E^2/\Lambda_M^2)\,, \tag{7}$$

for the vertex function, where p_E is the Euclidean Jacobi momentum. Here $\Lambda_{D_{s0}^*}$ is a size parameter, which parametrizes the distribution of D and K mesons inside the D_{s0}^* molecule, while $\Lambda_{D_{s1}}$ is the size parameter for the D_{s1} molecule. For simplicity we will use a universal scale parameter $\Lambda_M = \Lambda_{D_{s0}^*} = \Lambda_{D_{s1}}$, i.e. the same size for D_{s0}^* and D_{s1} mesons.

The coupling constants $g_{D_{s0}^*}$ and $g_{D_{s1}}$ are determined by the compositeness condition [32, 33], which implies that the renormalization constant of the hadron wave function is set equal to zero:

$$Z_{D_{s0}^*} \;=\; 1 - \Sigma'_{D_{s0}^*}(m^2_{D_{s0}^*}) = 0\,, \tag{8}$$

$$Z_{D_{s1}} \;=\; 1 - \Sigma'_{D_{s1}}(m^2_{D_{s1}}) = 0\,. \tag{9}$$

Here, $\Sigma'_{D_{s0}^*}(m^2_{D_{s0}^*}) = g^2_{D_{s0}^*}\,\Pi'_{D_{s0}^*}(m^2_{D_{s0}^*})$ is the derivative of the D_{s0}^* meson mass operator. In the case of the D_{s1} meson we have $\Sigma'_{D_{s1}}(m^2_{D_{s1}}) = g^2_{D_{s1}}\Pi'_{D_{s1}}(m^2_{D_{s1}})$, which is the derivative of the transverse part of its mass operator $\Sigma^{\mu\nu}_{D_{s1}}$, conventionally split into transverse $\Sigma_{D_{s1}}$ and longitudinal $\Sigma^L_{D_{s1}}$ parts as:

$$\Sigma^{\mu\nu}_{D_{s1}}(p) = g_\perp^{\mu\nu}\Sigma_{D_{s1}}(p^2) + \frac{p^\mu p^\nu}{p^2}\Sigma^L_{D_{s1}}(p^2)\,, \tag{10}$$

where

$$g_\perp^{\mu\nu} = g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\,, \qquad g_\perp^{\mu\nu}p_\mu = 0\,. \tag{11}$$

The mass operators of the D_{s0}^* and D_{s1} mesons are described by the diagram of Fig.1.

Following Eqs. (8) and (9) the coupling constants $g_{_{D_{s0}^*}}$ and $g_{_{D_{s1}}}$ can be expressed in the form:

$$\frac{1}{g^2_{_{D_{s0}^*}}} \;=\; \frac{2}{(4\pi\Lambda_M)^2}\int\limits_0^1 dx \int\limits_0^\infty \frac{d\alpha\,\alpha\,P_0(\alpha,x)}{(1+\alpha)^3}\,\left[-\frac{d}{dz_0}\tilde{\Phi}^2_M(z_0)\right], \tag{12}$$

$$\frac{1}{g^2_{_{D_{s1}}}} \;=\; \frac{2}{(4\pi\Lambda_M)^2}\int\limits_0^1 dx \int\limits_0^\infty \frac{d\alpha\,\alpha\,P_1(\alpha,x)}{(1+\alpha)^3}\,\left[\frac{1}{2\mu^2_{D^*}(1+\alpha)} - \frac{d}{dz_1}\right]\tilde{\Phi}^2_M(z_1) \tag{13}$$

where

$$
\begin{aligned}
P_0(\alpha,x) &= \alpha^2 x(1-x) + w^2_{_{DK}}\alpha x + w^2_{_{KD}}\alpha(1-x)\,, \\
P_1(\alpha,x) &= \alpha^2 x(1-x) + w^2_{_{D^*K}}\alpha x + w^2_{_{KD^*}}\alpha(1-x)\,, \\
z_0 &= \mu_D^2\alpha x + \mu_K^2\alpha(1-x) - \frac{P_0(\alpha,x)}{1+\alpha}\,\mu^2_{D^*_{s0}}\,, \\
z_1 &= \mu_{D^*}^2\alpha x + \mu_K^2\alpha(1-x) - \frac{P_1(\alpha,x)}{1+\alpha}\,\mu^2_{D_{s1}}\,, \\
\mu_M &= \frac{m_M}{\Lambda_M}\,.
\end{aligned}
\tag{14}
$$

Above expressions are valid for any functional form of the correlation function $\widetilde{\Phi}_M$.

Let us note, that the compositeness condition of the type (8), (9) was originally applied to the study of the deuteron as a bound state of proton and neutron [32]. Then this condition was extensively used in low-energy hadron phenomenology as the master equation for the treatment of mesons and baryons as bound states of light and heavy constituent quarks [33]-[35]. In Refs. [26, 36, 37] this condition was used in the application to glueballs as bound states of gluons and light and heavy scalar mesons as hadronic molecules.

III. LEPTONIC DECAY CONSTANTS $f_{D^*_{s0}}$ AND $f_{D_{s1}}$

The decay constants of the scalar D^*_{s0} and axial D_{s1} mesons are defined by

$$
\langle 0|\bar{s}O_\mu c|D^{*\,+}_{s0}(p)\rangle = p_\mu f_{_{D^*_{s0}}}\,, \tag{15}
$$

$$
\langle 0|\bar{s}O_\mu c|D^{+}_{s1}(p,\epsilon)\rangle = \epsilon_\mu(p)\, m_{D_{s1}} f_{_{D_{s1}}}\,, \tag{16}
$$

where $O_\mu = \gamma_\mu(1-\gamma_5)$. Note, parity symmetry implies that only the vector component of the weak $V-A$ current contributes to the transition $D^{*+}_{s0} \to W^+$, while for the transition $D^+_{s1} \to W^+$ only the axial component is present. The one-loop meson diagrams describing the matrix elements of Eqs. (15), (16) are given in Figs.2a and 2b. In other words, adopting the molecular picture for the D^*_{s0} and D_{s1} mesons we need to evaluate the two-point DK loop diagram describing the transition of the D^*_{s0} meson to the vector current and the two-point D^*K loop diagram corresponds to the transition of the D_{s1} meson to the axial current. The coupling of the weak $c \to s$ flavor-changing vector current (FCVC) V_μ to the DK pair and the weak $c \to s$ flavor-changing axial current (FCAC) A_μ to the D^*K pair can be extracted from data. In particular, the couplings are extracted by deriving the effective Lagrangians from the matrix elements for the $D \to K^{(*)}\ell\bar{\nu}_\ell$ semileptonic transitions. First, the matrix element for the $D \to K\ell\bar{\nu}_\ell$ transition [38] at zero momentum transfer $q = p - p' \to 0$ is approximated by

$$
\langle K(p')|V^\mu(0)|D(p)\rangle \simeq f_+^{DK}(0)(p+p')^\mu\,. \tag{17}
$$

Therefore, the effective Lagrangian describing the coupling of D and K mesons with the weak vector current is given by

$$
\mathcal{L}_{VDK}(x) = f_+^{DK}(0)\, V^\mu(x)\, D(x)\, i\partial^-_\mu\, K(x) + \text{H.c.} \tag{18}
$$

where $A\partial^-_\mu B = (\partial_\mu A)B - A\partial_\mu B$.

The effective coupling of the weak axial current A_μ with D^* and K mesons can be related to the corresponding coupling with D and K^* mesons on the basis of SU(4) flavor symmetry arguments. Both matrix elements for the semileptonic transitions $D^* \to K\ell\bar{\nu}_\ell$ and $D \to K^*\ell\bar{\nu}_\ell$ semileptonic transitions at zero momentum transfer can be approximately written as:

$$
\langle K(p')|A^\mu(0)|D^*(p,\epsilon)\rangle \;\simeq\; \epsilon^\mu(m_{D^*}+m_K)A_1^{D^*K}(0)\,, \tag{19}
$$

$$
\langle K^*(p',\epsilon')|A^\mu(0)|D(p)\rangle \;\simeq\; \epsilon'^{*\mu}(m_D+m_{K^*})A_1^{DK^*}(0)\,. \tag{20}
$$

At present no experimental information on the axial form factor $A_1^{D^*K}(0)$ is available, while the form factor $A_1^{DK^*}(0)$ is known, although with significant uncertainties. SU(4) symmetry requires that the axial form factors $A_1^{D^*K}(0)$ and $A_1^{DK^*}(0)$ satisfy the relation:

$$
(m_{D^*}+m_K)A_1^{D^*K}(0) \;\equiv\; (m_D+m_{K^*})A_1^{DK^*}(0)\,, \tag{21}
$$

which in turn allows to express the unknown form factor $A_1^{D^*K}(0)$ by $A_1^{DK^*}(0)$. The effective Lagrangian, describing the coupling of D^*K and DK^* meson pairs to the weak $c \to s$ FCAC is then given as:

$$\mathcal{L}_{AD^*K}(x) + \mathcal{L}_{ADK^*}(x) = (m_D + m_{K^*})\, A_1^{DK^*}(0)\, A^\mu(x) \biggl\{ D^*_\mu(x) K(x) + D(x) K^*_\mu(x) \biggr\} + \text{H.c.}\,. \tag{22}$$

In the following we calculate the leptonic decay constants $f_{D^*_{s0}}$ and $f_{D_{s1}}$ based on the effective interaction Lagrangian $\mathcal{L}_{\rm eff}$, which includes the couplings of D^*_{s0}, D_{s1} mesons and the weak currents (vector V^μ and axial A^μ) with DK and D^*K meson pairs:

$$\mathcal{L}_{\rm eff}(x) = \mathcal{L}_{D^*_{s0}}(x) + \mathcal{L}_{D_{s1}}(x) + \mathcal{L}_{VDK}(x) + \mathcal{L}_{AD^*K}(x)\,. \tag{23}$$

The corresponding diagrams are given in Fig.2. After a straightforward evaluation we obtain the following analytical expressions for the leptonic decay constants:

$$f_{D^*_{s0}} = \frac{g_{D^*_{s0}}}{8\pi^2}\, f_+^{DK}(0) \int\limits_0^1 dx \int\limits_0^\infty \frac{d\alpha\, \alpha}{(1+\alpha)^2}\, \tilde\Phi_M(z_0)\, \biggl[1 - \frac{2(w_{KD} + \alpha x)}{1+\alpha} \biggr]\,, \tag{24}$$

$$f_{D_{s1}} = \frac{g_{D_{s1}}}{8\pi^2}\, \frac{m_D + m_{K^*}}{m_{D_{s1}}}\, A_1^{DK^*}(0) \int\limits_0^1 dx \int\limits_0^\infty \frac{d\alpha\, \alpha}{(1+\alpha)^2}\, \tilde\Phi_M(z_1)\, \biggl[1 + \frac{\alpha}{4\mu^2_{D^*}} \frac{dz_1}{d\alpha} \biggr]\,, \tag{25}$$

where the coupling constants $g_{D^*_{s0}}$ and $g_{D_{s1}}$ are defined by Eqs. (12) and (13). We have only one model parameter in our calculations - the scale parameter Λ_M which was previously fixed to the value of $\Lambda_M = 2$ GeV [26, 27] from strong and radiative decays of D^*_{s0} and D_{s1} mesons. Further quantities, entering in Eqs. (24) and (25), are chosen as follows. The masses of mesons we take from the PDG [10]

$$\begin{aligned} & m_{D^*_{s0}} = 2.3173 \text{ GeV}\,, && m_{D_{s1}} = 2.4589 \text{ GeV}\,, \\ & m_K \equiv m_{K^\pm} = 493.677 \text{ MeV}\,, && m_{K^*} \equiv m_{K^{*\pm}} = 891.66 \text{ MeV}\,, \\ & m_D \equiv m_{D^\pm} = 1.8693 \text{ GeV}\,, && m_{D^*} \equiv m_{D^{*\pm}} = 2.010 \text{ GeV}\,. \end{aligned} \tag{26}$$

For the values of the $D \to K^{(*)}\ell\bar\nu_\ell$ semileptonic form factors at zero recoil, entering in our calculation, we use the world average data [39, 40, 41] of:

$$f_+^{DK}(0) = 0.75 \pm 0.05\,, \qquad A_1^{DK^*}(0) = 0.65 \pm 0.05\,. \tag{27}$$

Using this input we get the following results for $f_{D^*_{s0}}$ and $f_{D_{s1}}$:

$$f_{D^*_{s0}} = 67.1 \pm 4.5 \text{ MeV}\,, \qquad f_{D_{s1}} = 144.5 \pm 11.1 \text{ MeV}\,. \tag{28}$$

In Table 1 we summarize the present results for $f_{D^*_{s0}}$ and $f_{D_{s1}}$ obtained in different approaches (either on the basis of hadronic models or from the analysis of experimental data on two-body B-meson decays). Our results are in agreement with the predictions of Refs. [12, 17, 18, 20], especially with the lower limits derived from an analysis of the branching ratios of $B \to D^{(*)} D^*_{s0}(D_{s1})$ decays [18, 20].

At this point it is worth discussing the heavy quark limit (HQL) for the leptonic decay constants $f_{D^*_{s0}}$ and $f_{D_{s1}}$, where the masses of D, D^*, D^*_{s0} and D_{s1} mesons together with the charm quark mass m_c approach infinity. In the HQL the $D^{(*)}$ mesons in the $D^*_{s0}(D_{s1})$ hadronic molecules move to the center of mass and are surrounded by a light K meson in analogy with the heavy-light $Q\bar q$ mesons. It is known (see e.g. discussion in Refs. [17, 18]) that in the two-quark picture for the D^*_{s0}, D_{s1} mesons HQL leads to degenerate values for these couplings

$$f_{D^*_{s0}} \equiv f_{D_{s1}} \sim \frac{1}{\sqrt{m_c}}\,. \tag{29}$$

In the present molecular approach we can also guarantee that the couplings are degenerate in the HQL, but the scaling law is different from the $1/\sqrt{m_c}$ behavior. In a first step we apply the HQL to the coupling constants $g_{D^*_{s0}}$ and $g_{D_{s1}}$, which are degenerate:

$$\frac{1}{g^2_{D^*_{s0}}} \equiv \frac{1}{g^2_{D_{s1}}} = \frac{I_0}{(4\pi m_c)^2}\,, \tag{30}$$

$$I_0 = \int\limits_0^\infty \frac{d\alpha}{1+\alpha}\, \Phi^2_M(\mu^2_K \alpha)\,.$$

The structure integrals $\int\limits_0^1 dx \int\limits_0^\infty d\alpha \cdots$ entering in Eqs. (24) and (25) are also degenerate and equal

$$I_1 = \frac{\Lambda_M}{m_c} \int\limits_0^\infty \frac{d\alpha\,\sqrt{\alpha}}{1+\alpha}\,\Phi_M^2(\mu_K^2\alpha)\,. \tag{31}$$

Finally, the leptonic decay constants $f_{D^*_{s0}}$ and $f_{D_{s1}}$ in the HQL are given by:

$$f_{D^*_{s0}} = \frac{\Lambda_M}{2\pi}\, g^{hql}_{VDK}\, \frac{I_1}{\sqrt{I_0}}\,, \tag{32}$$

$$f_{D_{s1}} = \frac{\Lambda_M}{2\pi}\, g^{hql}_{AD^*K}\, \frac{I_1}{\sqrt{I_0}}\,, \tag{33}$$

where we introduced the effective VDK and AD^*K couplings in the HQL: g^{hql}_{VDK} and $g^{hql}_{AD^*K}$. From Eqs. (32) and (33) we deduce that in the HQL $f_{D^*_{s0}}$ and $f_{D_{s1}}$ in the HQL limit do not depend on m_c at all, unlike as in Eq. (29), and are degenerate for $g^{hql}_{VDK} = g^{hql}_{AD^*K}$. The latter condition can be eventually fulfilled, e.g. at finite masses (see Eqs. (24) and (25)) the ratio g_{AD^*K}/g_{VDK} is close to 1:

$$\frac{g_{AD^*K}}{g_{VDK}} = \frac{m_D + m_{K^*}}{m_{D_{s1}}}\,\frac{A_1^{DK^*}(0)}{f_+^{DK}(0)} \simeq 1\,. \tag{34}$$

To give an estimate for the absolute values of the leptonic decay constants $f_{D^*_{s0}}$ and $f_{D_{s1}}$ in the HQL we vary the coupling constants $g^{hql}_{VDK} = g^{hql}_{AD^*K}$ in the region 0.75 ± 0.25. The corresponding result is:

$$f_{D^*_{s0}} = f_{D_{s1}} = 205.2 \pm 68.4 \text{ MeV}\,. \tag{35}$$

The following comment is in order. As was already stressed in Ref. [18] the HQL does not give a reasonable approximation for the P-wave D_{sJ} meson system. According to the data on the two-body decays $B \to D^{(*)} D^*_{s0}(D_{s1})$ the physical value of the decay constant $f_{D_{s1}}$ should be about twice as large as $f_{D^*_{s0}}$ [18, 20].

IV. WEAK TWO-BODY DECAYS $B \to D^{(*)} D^*_{s0}(D_{s1})$

In this section we give the predictions for the branching ratios of $B \to D^{(*)} D^*_{s0}(D_{s1})$ decays. For this purpose we use leptonic decay constants $f_{D^*_{s0}}$, $f_{D_{s1}}$ and, in addition, model-independent results for the form factors of $B \to D^{(*)}\ell\bar\nu_\ell$ transitions obtained by Caprini, Lellouch and Neubert (CLN) [28]. Latter derivations are based on heavy quark spin symmetry, dispersive constraints, including short-distance and power corrections. Note, that in Ref. [18] the authors already used the CLN results in their analysis of two-body $B \to D D^*_{s0}(D_{s1})$ transitions, restricting to modes containing the pseudoscalar meson D in the final state.

Working with the factorization approximation we first write down the factorizable amplitudes for the four decay modes $\bar B^0 \to D^+ D^{*-}_{s0}$, $D^+ D^-_{s1}$, $D^{*+} D^{*-}_{s0}$ and $D^{*+} D^-_{s1}$ as expressed in terms of matrix elements of the semileptonic $B \to D^{(*)}\ell\bar\nu_\ell$ and the leptonic D^*_{s0}, D_{s1} transitions and (see details in Refs. [17, 18, 20]):

$$M(\bar B^0 \to D^+ D^{*-}_{s0}) = G_{\rm eff}\langle D^{*-}_{s0}(q)|\bar s O_\mu c|0\rangle\, \langle D^+(p')|\bar c O^\mu b|\bar B^0(p)\rangle\,, \tag{36}$$

$$M(\bar B^0 \to D^{*+} D^{*-}_{s0}) = G_{\rm eff}\langle D^{*-}_{s0}(q)|\bar s O_\mu c|0\rangle\, \langle D^{*+}(p',\epsilon')|\bar c O^\mu b|\bar B^0(p)\rangle\,, \tag{37}$$

$$M(\bar B^0 \to D^+ D^-_{s1}) = G_{\rm eff}\langle D^-_{s1}(q,\epsilon)|\bar s O_\mu c|0\rangle\, \langle D^+(p)|\bar c O^\mu b|\bar B^0(p)\rangle\,, \tag{38}$$

$$M(\bar B^0 \to D^{*+} D^-_{s1}) = G_{\rm eff}\langle D^-_{s1}(q,\epsilon)|\bar s O_\mu c|0\rangle\, \langle D^{*+}(p',\epsilon')|\bar c O^\mu b|\bar B^0(p)\rangle\,. \tag{39}$$

Here

$$G_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{cb} V^*_{cs} a_1 \tag{40}$$

and $a_1 = c_2 + c_1/N_c$ is the combination of the short-distance Wilson coefficients c_1 and c_2 [29]-[31]. Using the Wirbel-Stech-Bauer (WSB) decomposition [38, 42] of the hadronic $B \to D^{(*)}$ matrix elements

$$\langle D^+(p')|V^\mu|\bar B^0(p)\rangle = \left\{(p+p')^\mu - \frac{m_B^2 - m_D^2}{q^2} q^\mu\right\} F_1(q^2) + \frac{m_B^2 - m_D^2}{q^2} q^\mu F_0(q^2)\,, \tag{41}$$

$$
\begin{aligned}
\langle D^{*+}(p',\epsilon')|V^\mu|\bar{B}^0(p)\rangle &= \frac{2i\epsilon^{\mu\nu\alpha\beta}}{m_B+m_{D^*}}\,\epsilon'^*_\nu\,p'_\alpha\,p_\beta\,V(q^2)\,, & (42)\\
\langle D^{*+}(p',\epsilon')|A^\mu|\bar{B}^0(p)\rangle &= \epsilon'^{*\mu}(m_B+m_{D^*})A_1(q^2)\\
&- \frac{\epsilon'^*q}{m_B+m_{D^*}}(p+p')^\mu A_2(q^2)+2m_{D^*}q^\mu\frac{\epsilon'^*q}{q^2}\Big\{A_0(q^2)-A_3(q^2)\Big\}\,, & (43)
\end{aligned}
$$

where $V^\mu=\bar{c}\gamma^\mu b$, $A^\mu=\bar{c}\gamma^\mu\gamma^5 b$ and

$$
A_3(q^2)=\frac{m_B+m_{D^*}}{2m_{D^*}}A_1(q^2)-\frac{m_B-m_{D^*}}{2m_{D^*}}A_2(q^2) \qquad (44)
$$

we arrive at [17, 18, 20]:

$$
\begin{aligned}
M(\bar{B}^0\to D^+D^{*-}_{s0}) &= G_{\rm eff}\,f_{D^*_{s0}}\,(m_B^2-m_D^2)\,F_0(m^2_{D^*_{s0}})\,, & (45)\\
M(\bar{B}^0\to D^+D^{-}_{s1}) &= G_{\rm eff}\,f_{D_{s1}}\,m_{D_{s1}}\,\epsilon^*(p+p')\,F_1(m^2_{D_{s1}})\,, & (46)\\
M(\bar{B}^0\to D^{*+}D^{*-}_{s0}) &= 2\,G_{\rm eff}\,f_{D^*_{s0}}\,m_{D^*}\,\epsilon'^*p\,A_0(m^2_{D^*_{s0}})\,, & (47)\\
M(\bar{B}^0\to D^{*+}D^{-}_{s1}) &= G_{\rm eff}\,f_{D_{s1}}\,m_{D_{s1}}\Big\{\frac{2\,i\epsilon^{\mu\nu\alpha\beta}}{m_B+m_{D^*}}\,\epsilon^*_\mu\epsilon'^*_\nu\,p'_\alpha\,p_\beta\,V(m^2_{D_{s1}})\\
&+ \epsilon'^*\epsilon^*(m_B+m_{D^*})A_1(m^2_{D_{s1}})-\frac{2\,\epsilon'^*p\,\epsilon^*p}{m_B+m_{D^*}}A_2(m^2_{D_{s1}})\Big\}\,. & (48)
\end{aligned}
$$

In the following it is convenient to express the WSB set of form factors through a set of form factors $h_i(w)$ depending on the kinematical variable $w=v\cdot v'$, the scalar product of the four-velocities of $\bar{B}$ and $D^{(*)}$ mesons [28, 42, 43]:

$$
\begin{aligned}
\langle D(v')|V^\mu|\bar{B}(v)\rangle &= h_+(w)(v+v')^\mu+h_-(w)(v-v')^\mu\,, & (49)\\
\langle D^*(v',\epsilon')|V^\mu|\bar{B}(v)\rangle &= ih_V(w)\epsilon^{\mu\nu\alpha\beta}\,\epsilon'^*_\nu\,v'_\alpha\,v_\beta\,, & (50)\\
\langle D^*(v',\epsilon')|A^\mu|\bar{B}(v)\rangle &= h_{A_1}(w)\,(w+1)\epsilon'^{*\mu}-\Big[h_{A_2}(w)v^\mu+h_{A_3}(w)v'^\mu\Big]\epsilon'^*v\,, & (51)
\end{aligned}
$$

where the meson states $|M(v)\rangle$ obey the mass-independent normalization condition

$$
\langle M(v')|M(v)\rangle = 2v^0(2\pi)^3\delta^3(\mathbf{p}-\mathbf{p}')\,, \qquad (52)
$$

instead of the relativistic one used before:

$$
\langle M(p')|M(p)\rangle = 2p^0(2\pi)^3\delta^3(\mathbf{p}-\mathbf{p}')\,. \qquad (53)
$$

The WSB form factors are expressed in terms of the $h_i(w)$ form factors as:

$$
\begin{aligned}
F_1(q^2) &= \frac{1+r}{2\sqrt{r}}\,V_1(w)\,, & (54)\\
F_0(q^2) &= \frac{2\sqrt{r}}{1+r}\,\frac{w+1}{2}\,V_1(w)\,R(w)\,, & (55)\\
V(q^2) &= \frac{1+r^*}{2\sqrt{r^*}}\,h_{A_1}(w)\,R_1(w)\,, & (56)\\
A_1(q^2) &= \frac{2\sqrt{r^*}}{1+r^*}\,\frac{w+1}{2}\,h_{A_1}(w)\,, & (57)\\
A_2(q^2) &= \frac{1+r^*}{2\sqrt{r^*}}\,h_{A_1}(w)\,R_2(w)\,, & (58)\\
A_0(q^2) &= \frac{1}{2\sqrt{r^*}}\,h_{A_1}(w)\,R_0(w)\,, & (59)
\end{aligned}
$$

where $r=m_D/m_B$, $r^*=m_{D^*}/m_B$ and

$$
V_1(w)=G(w)=h_+(w)-\frac{1-r}{1+r}h_-(w)\,. \qquad (60)
$$

Here we use the well-known form factor ratios $R_1(w)$ and $R_2(w)$ [28, 42]:

$$R_1(w) = \frac{h_V(w)}{h_{A_1}(w)}\,, \qquad\qquad R_2(w) = \frac{h_{A_3}(w) + r^* h_{A_2}(w)}{h_{A_1}(w)}\,. \tag{61}$$

In the literature the form factor $V_1(w)$ is also denoted as $G(w)$. In order to express all $h_i(w)$ form factors, completely defining the $B \to D^{(*)}$ transitions, through the two form factors $V_1(w)$ and $h_{A_1}(w)$ we introduce the additional ratios $R(w)$, $R_2^*(w)$:

$$R(w) = \frac{h_+(w) - \dfrac{1+r}{1-r}\dfrac{w-1}{w+1}h_-(w)}{h_+(w) - \dfrac{1-r}{1+r}h_-(w)}\,, \qquad\qquad R_2^*(w) = \frac{h_{A_3}(w) - r^* h_{A_2}(w)}{h_{A_1}(w)}\,. \tag{62}$$

and $R_0(w)$, which is just the combination of $R_2(w)$ and $R_2^*(w)$:

$$R_0(w) = (w+1)[1 - R_2^*(w)] + \frac{(1+r^*)^2}{2r^*}\left[R_2^*(w) - \frac{1-r^*}{1+r^*}R_2(w)\right]. \tag{63}$$

For the functions $V_1(w) = G(w)$, $h_{A_1}(w)$, $R_0(w)$, $R_1(w)$, $R_2(w)$ and $R_2^*(w)$ we use the model-independent results derived by Caprini, Lellouch and Neubert (CLN) [28]:

$$\frac{G(w)}{G(1)} = 1 - 8\rho_G^2 z + (51\rho_G^2 - 10)z^2 - (252\rho_G^2 - 84)z^3\,, \tag{64}$$
$$\frac{h_{A_1}(w)}{h_{A_1}(1)} = 1 - 8\rho_{h_{A_1}}^2 z + (53\rho_{h_{A_1}}^2 - 15)z^2 - (231\rho_{h_{A_1}}^2 - 91)z^3\,, \tag{65}$$
$$R(w) = 1.004 - 0.007(w-1) + 0.002(w-1)^2\,, \tag{66}$$
$$R_1(w) = 1.27 - 0.12(w-1) + 0.05(w-1)^2\,, \tag{67}$$
$$R_2(w) = 0.80 + 0.11(w-1) - 0.06(w-1)^2\,, \tag{68}$$
$$R_2^*(w) = 1.15 - 0.07(w-1) - 0.11(w-1)^2\,, \tag{69}$$

where

$$z = \frac{\sqrt{w+1} - \sqrt{2}}{\sqrt{w+1} + \sqrt{2}}\,. \tag{70}$$

In the derivation of the expressions for the ratios $R_0(w)$ and $R_2^*(w)$ we used the results of Ref. [28] (see Appendix). Note, that $R(w)$ is very close to 1 for w in the interval $1 \le w \le w_{\rm max} = (m_B^2 + m_D^2)/(2m_B m_D)$.

The two-body decay widths $\Gamma(\bar B \to D^{(*)} D_{s0}^{*-})$ and $\Gamma(\bar B \to D^{(*)} D_{s1}^-)$, given in terms of the CLN form factors, are expressed by the formulas (see e.g. Ref. [18]):

$$\Gamma(\bar B \to D D_{s0}^{*-}) = \frac{G_{\rm eff}^2}{8\pi m_B}\, f_{D_{s0}^*}^2\, (m_B - m_D)^2\, m_D^2\, (w_1+1)^2 \sqrt{w_1^2 - 1}\, [G(w_1)\, R(w_1)]^2 \tag{71}$$
$$\Gamma(\bar B \to D D_{s1}^-) = \frac{G_{\rm eff}^2}{8\pi m_B}\, f_{D_{s1}}^2\, (m_B + m_D)^2\, m_D^2\, (w_2^2 - 1)^{3/2}\, G^2(w_2) \tag{72}$$
$$\Gamma(\bar B \to D^* D_{s0}^{*-}) = \frac{G_{\rm eff}^2}{8\pi m_B}\, f_{D_{s0}^*}^2\, m_B^2\, m_{D^*}^2\, (w_3^2 - 1)^{3/2}\, [h_{A_1}(w_3)\, R_0(w_3)]^2 \tag{73}$$
$$\Gamma(\bar B \to D^* D_{s1}^-) = \frac{G_{\rm eff}^2}{8\pi m_B}\, f_{D_{s1}}^2\, (m_B - m_{D^*})^2\, m_{D^*}^2\, (w_4+1)^2 \sqrt{w_4^2 - 1}\, h_{A_1}^2(w_4)\, \beta_{h_{A_1}}(w_4)\,, \tag{74}$$

where the kinematical variables w_i are defined as follows:

$$w_1 = \frac{m_B^2 + m_D^2 - m_{D_{s0}^*}^2}{2m_B m_D}\,, \qquad w_2 = \frac{m_B^2 + m_D^2 - m_{D_{s1}}^2}{2m_B m_D}\,,$$
$$w_3 = \frac{m_B^2 + m_{D^*}^2 - m_{D_{s0}^*}^2}{2m_B m_{D^*}}\,, \qquad w_4 = \frac{m_B^2 + m_{D^*}^2 - m_{D_{s1}}^2}{2m_B m_{D^*}}\,, \tag{75}$$

and

$$\beta_{h_{A_1}}(w) = 2\frac{1-2wr^* + r^{*2}}{(1-r^*)^2}\left[1 + \frac{w-1}{w+1}R_1^2(w)\right] + \left[1 + \frac{w-1}{1-r^*}(1-R_2(w))\right]^2 . \tag{76}$$

Note that the product $h^2_{A_1}(w)\,\beta_{h_{A_1}}(w)$ defines the well-known function $F(w)$ [28, 42], which governs the semileptonic transition $\bar{B} \to D^*\ell\bar{\nu}_\ell$ (see e.g. Eq.(35) in Ref. [28]):

$$h^2_{A_1}(w)\,\beta_{h_{A_1}}(w) \equiv F^2(w)\left[1 + \frac{4w}{w+1}\,\frac{1-2wr^*+r^{*2}}{(1-r^*)^2}\right] . \tag{77}$$

In the numerical calculation we use the value for the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $V_{cs} = 0.97296 \pm 0.00024$ from the 2006 global fit [10, 41] and the 2006 averaged values from the Heavy Flavor Averaging Group (HFAG) [44]:

$$|V_{cb}|\,F(1) = (36.2 \pm 0.8) \times 10^{-3}\,, \qquad \rho_F^2 = 1.19 \pm 0.06\,, \tag{78}$$

$$|V_{cb}|\,G(1) = (42.4 \pm 4.4) \times 10^{-3}\,, \qquad \rho_G^2 = 1.17 \pm 0.18\,, \tag{79}$$

where the normalization of $F(w)$ and its slope ρ_F^2 at $w = 1$ are related to the characteristics of the $h_{A_1}(w)$ form factor as [28, 42]:

$$F(1) \equiv h_{A_1}(1)\,, \qquad \rho_F^2 \simeq \rho^2_{h_{A1}} - 0.21\,. \tag{80}$$

For the parameter a_1 we use the value of 1.05 from Ref. [31]. A detailed discussion concerning the choice of a_1 can be found in Ref. [30].

In Table 2 we present our predictions for the branching ratios of two-body decays $B \to D^{(*)}D^*_{s0}(D_{s1})$. For the data we use the averaged lower limits from PDG [10] and in addition for the modes with $D_{s1}(2460)$ meson in the final state the direct results of the BABAR Collaboration [9]. Our predictions are in good agreement with the experimental data except for the marginal situation in the case of $\bar{B}^0 \to D^{*-}_{s0}D^{*+}$ decay, where our prediction is slightly lower than the experimental limit. In Table 3 we present the results for the ratios $\Gamma(B \to D^*D^*_{s0})/\Gamma(B \to DD^*_{s0})$ and $\Gamma(B \to D^*D_{s1})/\Gamma(B \to DD_{s1})$. We also use the compilation of experimental data and theoretical results within the covariant light-front (CLF) approach [17] summarized in Table 10 of Ref. [20]. Here, our predictions are in good agreement with the existing experimental data. In comparison with the CLF approach our predictions are lower, although not significantly.

V. CONCLUSION

We considered the new charmed-strange mesons $D^*_{s0}(2317)$ and $D_{s1}(2460)$ as hadronic molecules, that is DK and D^*K bound states, respectively. Using an effective Lagrangian approach describing the coupling of $D^*_{s0}(2317)$ and $D_{s1}(2460)$ states with their constituents we determined the leptonic decay constants $f_{D^*_{s0}}$ and $f_{D_{s1}}$. Then we presented a detailed analysis of two-body bottom mesons decays $B \to D^{(*)}D^*_{s0}(D_{s1})$ using the factorization hypothesis and model-independent results for the form factors of the $B \to D^{(*)}\ell\bar{\nu}_\ell$ transitions obtained by Caprini, Lellouch and Neubert (CLN) [28]. The decay widths are derived in terms of the CLN form factors. Calculated branching ratios for $B \to D^{(*)}D^*_{s0}(D_{s1})$ decays are in good agreement with experimental results (or the lower limits), supporting a possible interpretation of $D^*_{s0}(2317)$ and $D_{s1}(2460)$ as hadronic molecules.

Acknowledgments

This work was supported by the DFG under contracts FA67/31-1 and GRK683 and by CONICYT (Chile) under grants FONDECYT No.1030244 and PBCT/No.285/2006. This research is also part of the EU Integrated Infrastructure Initiative Hadronphysics project under contract number RII3-CT-2004-506078 and President grant of Russia "Scientific Schools" No. 5103.2006.2.

[1] J. L. Rosner, Phys. Rev. D **74**, 076006 (2006) [arXiv:hep-ph/0608102].

[2] P. Colangelo, F. De Fazio and R. Ferrandes, Mod. Phys. Lett. A **19**, 2083 (2004) [arXiv:hep-ph/0407137].
[3] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **90**, 242001 (2003) [arXiv:hep-ex/0304021].
[4] D. Besson *et al.* [CLEO Collaboration], Phys. Rev. D **68**, 032002 (2003) [arXiv:hep-ex/0305100].
[5] K. Abe *et al.*, Phys. Rev. Lett. **92** (2004) 012002 [arXiv:hep-ex/0307052].
[6] P. Krokovny *et al.* [Belle Collaboration], Phys. Rev. Lett. **91**, 262002 (2003) [arXiv:hep-ex/0308019].
[7] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **93**, 181801 (2004) [arXiv:hep-ex/0408041].
[8] A. Drutskoy, J. Phys. Conf. Ser. **9**, 115 (2005) [arXiv:hep-ex/0412070].
[9] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. D **74**, 031103 (2006) [arXiv:hep-ex/0605036].
[10] W. M. Yao *et al.* [Particle Data Group], J. Phys. G **33**, 1 (2006).
[11] A. Le Yaouanc, L. Oliver, O. Pene, J. C. Raynal and V. Morenas, Phys. Lett. B **520**, 59 (2001) [arXiv:hep-ph/0107047].
[12] H. Y. Cheng and W. S. Hou, Phys. Lett. B **566**, 193 (2003) [arXiv:hep-ph/0305038].
[13] H. Y. Cheng, Phys. Rev. D **68**, 094005 (2003) [arXiv:hep-ph/0307168].
[14] R. C. Hsieh, C. H. Chen and C. Q. Geng, Mod. Phys. Lett. A **19**, 597 (2004) [arXiv:hep-ph/0312232].
[15] A. Datta and P. J. O'Donnell, Phys. Lett. B **572**, 164 (2003) [arXiv:hep-ph/0307106].
[16] A. Datta, Int. J. Mod. Phys. A **19**, 5501 (2004) [arXiv:hep-ph/0407330].
[17] H. Y. Cheng, C. K. Chua and C. W. Hwang, Phys. Rev. D **69**, 074025 (2004) [arXiv:hep-ph/0310359].
[18] D. S. Hwang and D. W. Kim, Phys. Lett. B **606**, 116 (2005) [arXiv:hep-ph/0410301].
[19] C. E. Thomas, Phys. Rev. D **73**, 054016 (2006) [arXiv:hep-ph/0511169].
[20] H. Y. Cheng and C. K. Chua, Phys. Rev. D **74**, 034020 (2006) [arXiv:hep-ph/0605073].
[21] P. Colangelo, G. Nardulli, A. A. Ovchinnikov and N. Paver, Phys. Lett. B **269**, 201 (1991).
[22] S. Veseli and I. Dunietz, Phys. Rev. D **54**, 6803 (1996) [arXiv:hep-ph/9607293].
[23] P. Colangelo, F. De Fazio and A. Ozpineci, Phys. Rev. D **72**, 074004 (2005) [arXiv:hep-ph/0505195].
[24] F. Jugeau, A. Le Yaouanc, L. Oliver and J. C. Raynal, Phys. Rev. D **72**, 094010 (2005) [arXiv:hep-ph/0504206].
[25] G. Herdoiza, C. McNeile and C. Michael [UKQCD Collaboration], Phys. Rev. D **74**, 014510 (2006) [arXiv:hep-lat/0604001].
[26] A. Faessler, T. Gutsche, V. E. Lyubovitskij and Y. L. Ma, arXiv:0705.0254 [hep-ph].
[27] A. Faessler, Th. Gutsche, V. E. Lyubovitskij, Y. L. Ma, in preparation.
[28] I. Caprini, L. Lellouch and M. Neubert, Nucl. Phys. B **530**, 153 (1998) [arXiv:hep-ph/9712417].
[29] G. Buchalla, A. J. Buras and M. E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996) [arXiv:hep-ph/9512380].
[30] H. Y. Cheng and K. C. Yang, Phys. Rev. D **59**, 092004 (1999) [arXiv:hep-ph/9811249].
[31] Z. Luo and J. L. Rosner, Phys. Rev. D **64**, 094001 (2001) [arXiv:hep-ph/0101089].
[32] S. Weinberg, Phys. Rev. **130**, 776 (1963); A. Salam, Nuovo Cim. **25**, 224 (1962); K. Hayashi, M. Hirayama, T. Muta, N. Seto and T. Shirafuji, Fortsch. Phys. **15**, 625 (1967).
[33] G. V. Efimov and M. A. Ivanov, *The Quark Confinement Model of Hadrons*, (IOP Publishing, Bristol & Philadelphia, 1993).
[34] G. V. Efimov, M. A. Ivanov and V. E. Lyubovitskij, Few Body Syst. **6**, 17 (1989) [Acta Phys. Austriaca **6**, 17 (1989)]; I. V. Anikin, M. A. Ivanov, N. B. Kulimanova and V. E. Lyubovitskij, Z. Phys. C **65**, 681 (1995); M. A. Ivanov, M. P. Locher and V. E. Lyubovitskij, Few Body Syst. **21**, 131 (1996); M. A. Ivanov, V. E. Lyubovitskij, J. G. Körner and P. Kroll, Phys. Rev. D **56**, 348 (1997) [arXiv:hep-ph/9612463]; M. A. Ivanov and V. E. Lyubovitskij, Phys. Lett. B **408**, 435 (1997) [arXiv:hep-ph/9705423]; M. A. Ivanov, J. G. Körner and V. E. Lyubovitskij, Phys. Lett. B **448**, 143 (1999) [arXiv:hep-ph/9811370]; M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij and A. G. Rusetsky, Phys. Rev. D **60**, 094002 (1999) [arXiv:hep-ph/9904421]; M. A. Ivanov and P. Santorelli, Phys. Lett. B **456**, 248 (1999) [arXiv:hep-ph/9903446]; A. Faessler, T. Gutsche, M. A. Ivanov, J. G. Körner and V. E. Lyubovitskij, Phys. Lett. B **518**, 55 (2001) [arXiv:hep-ph/0107205]; A. Faessler, T. Gutsche, M. A. Ivanov, V. E. Lyubovitskij and P. Wang, Phys. Rev. D **68**, 014011 (2003) [arXiv:hep-ph/0304031]. M. A. Ivanov, J. G. Korner and P. Santorelli, Phys. Rev. D **73**, 054024 (2006) [arXiv:hep-ph/0602050]; A. Faessler, T. Gutsche, M. A. Ivanov, J. G. Korner, V. E. Lyubovitskij, D. Nicmorus and K. Pumsa-ard, Phys. Rev. D **73**, 094013 (2006) [arXiv:hep-ph/0602193]; A. Faessler, T. Gutsche, B. R. Holstein, V. E. Lyubovitskij, D. Nicmorus and K. Pumsa-ard, Phys. Rev. D **74**, 074010 (2006) [arXiv:hep-ph/0608015].
[35] Y. V. Burdanov, G. V. Efimov, S. N. Nedelko and S. A. Solunin, Phys. Rev. D **54**, 4483 (1996) [arXiv:hep-ph/9601344]; I. V. Anikin, A. E. Dorokhov and L. Tomio, Phys. Part. Nucl. **31**, 509 (2000) [Fiz. Elem. Chast. Atom. Yadra **31**, 1023 (2000)].
[36] J. V. Burdanov and G. V. Efimov, Phys. Rev. D **64**, 014001 (2001) [arXiv:hep-ph/0009027].
[37] V. Baru, J. Haidenbauer, C. Hanhart, Yu. Kalashnikova and A. E. Kudryavtsev, Phys. Lett. B **586**, 53 (2004) [arXiv:hep-ph/0308129]; C. Hanhart, Yu. S. Kalashnikova, A. E. Kudryavtsev and A. V. Nefediev, Phys. Rev. D **75**, 074015 (2007) [arXiv:hep-ph/0701214].
[38] M. Wirbel, B. Stech and M. Bauer, Z. Phys. C **29**, 637 (1985).
[39] J. M. Flynn and C. T. Sachrajda, Adv. Ser. Direct. High Energy Phys. **15**, 402 (1998) [arXiv:hep-lat/9710057].
[40] P. Ball, Phys. Lett. B **641**, 50 (2006) [arXiv:hep-ph/0608116].
[41] P. Zweber, arXiv:hep-ex/0701018.
[42] M. Neubert, Phys. Rept. **245**, 259 (1994) [arXiv:hep-ph/9306320].
[43] A. F. Falk and M. Neubert, Phys. Rev. D **47**, 2965 (1993) [arXiv:hep-ph/9209268]; Phys. Rev. D **47**, 2982 (1993) [arXiv:hep-ph/9209269].
[44] Heavy Flavor Averaging Group (HFAG), http://www.slac.stanford.edu/xorg/hfag/

Table 1. Leptonic decay constants $f_{D^*_{s0}}$ and $f_{D_{s1}}$.

Approach	$f_{D^*_{s0}}$ (MeV)	$f_{D_{s1}}$ (MeV)
Ref. [23]	225 ± 25	225 ± 25
Ref. [24]	206 ± 120	
Ref. [25]	200±50	
Ref. [21]	170 ± 20	247 ± 37
Ref. [14]	138±16	259±13
Ref. [22]	110±18	233±31
Ref. [18]	$> (74 \pm 11)/\lvert a_1\rvert$	$> (166 \pm 20)/\lvert a_1\rvert$
Ref. [17]	71	117
Ref. [17]	60 ± 13	150 ± 40
Ref. [20]	$>$ (58 - 86)/$\lvert a_1\rvert$	$>$ (90 - 228)/$\lvert a_1\rvert$
Ref. [12]	67±13	
Ref. [11]	44	41
Our results	67.1 ± 4.5	144.5 ± 11.1

Table 2. Branching ratios of $B \to D^{(*)} D^*_{s0}(D_{s1})$ decays (in units of 10^{-3}).

Mode	Data (averaged) [10]	BABAR [9]	Our results
$B^- \to D^{*-}_{s0} D^0$	$> 0.74^{+0.23}_{-0.19}$		1.03 ± 0.14
$\bar{B}^0 \to D^{*-}_{s0} D^+$	$> 0.97^{+0.41}_{-0.34}$		0.96 ± 0.13
$B^- \to D^-_{s1} D^0$	$> 1.4^{+0.6}_{-0.5}$	$4.3 \pm 1.6 \pm 1.3$	2.54 ± 0.39
$\bar{B}^0 \to D^-_{s1} D^+$	$> 2.0^{+0.6}_{-0.5}$	$2.6 \pm 1.5 \pm 0.74$	2.36 ± 0.36
$B^- \to D^{*-}_{s0} D^{*0}$	$> 0.9 \pm 0.6^{+0.4}_{-0.3}$		0.50 ± 0.07
$\bar{B}^0 \to D^{*-}_{s0} D^{*+}$	$> 1.5 \pm 0.4^{+0.5}_{-0.4}$		0.47 ± 0.06
$B^- \to D^-_{s1} D^{*0}$	$> 5.5 \pm 1.2^{+2.2}_{-1.6}7.6$	$11.2 \pm 2.6 \pm 2.0$	7.33 ± 1.12
$\bar{B}^0 \to D^-_{s1} D^{*+}$	$> 7.6 \pm 1.7^{+3.2}_{-2.4}$	$8.8 \pm 2.0 \pm 1.4$	6.85 ± 1.05

Table 3. Ratios $MD^*/MD = \Gamma(B \to MD^*)/\Gamma(B \to MD)$ for $M = D^*_{s0}, D_{s1}$.

Mode	Data [20]	CLF [20]	Our results
$D^{*-}_{s0} D^{*0}/D^{*-}_{s0} D^0$	0.91 ± 0.73	0.49	0.48
$D^{*-}_{s0} D^{*+}/D^{*-}_{s0} D^+$	0.59 ± 0.26	0.49	0.48
$D^-_{s1} D^{*0}/D^-_{s1} D^0$	3.4 ± 2.4	3.6	2.9
$D^-_{s1} D^{*+}/D^-_{s1} D^+$	2.6 ± 1.5	3.6	2.9

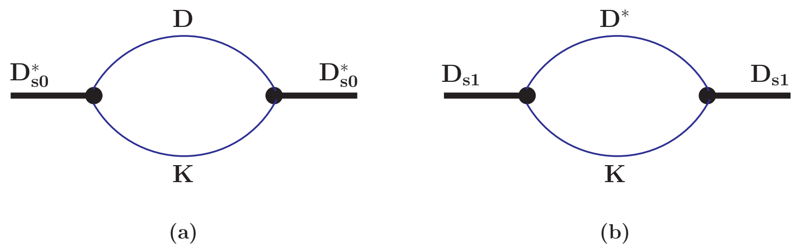

FIG. 1: Mass operators of $D_{s0}^*(2317)$ and $D_{s1}(2460)$ mesons.

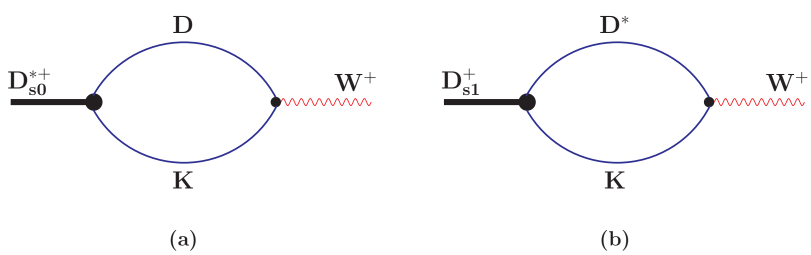

FIG. 2: Diagrams related to the leptonic decay constants of $D_{s0}^*(2317)$ and $D_{s1}(2460)$ mesons.